Modeling and solving optimization problems with decision diagrams

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👋 Hello and welcome

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A brief history optimization

Once upon a time in 1940 to now with many matrices

What are decision diagrams?

Overview of fundamentals, state-based search, hybrid solving

Decision diagrams in action

Opportunities DDs offer for vehicle routing, runtime, and more

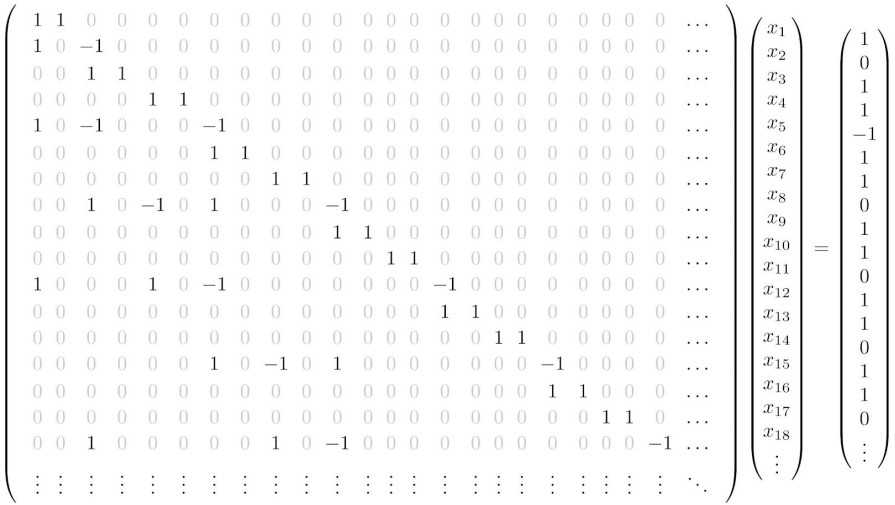
Q&A time

Ask questions in the chat or Q&A feature

A brief history of optimization

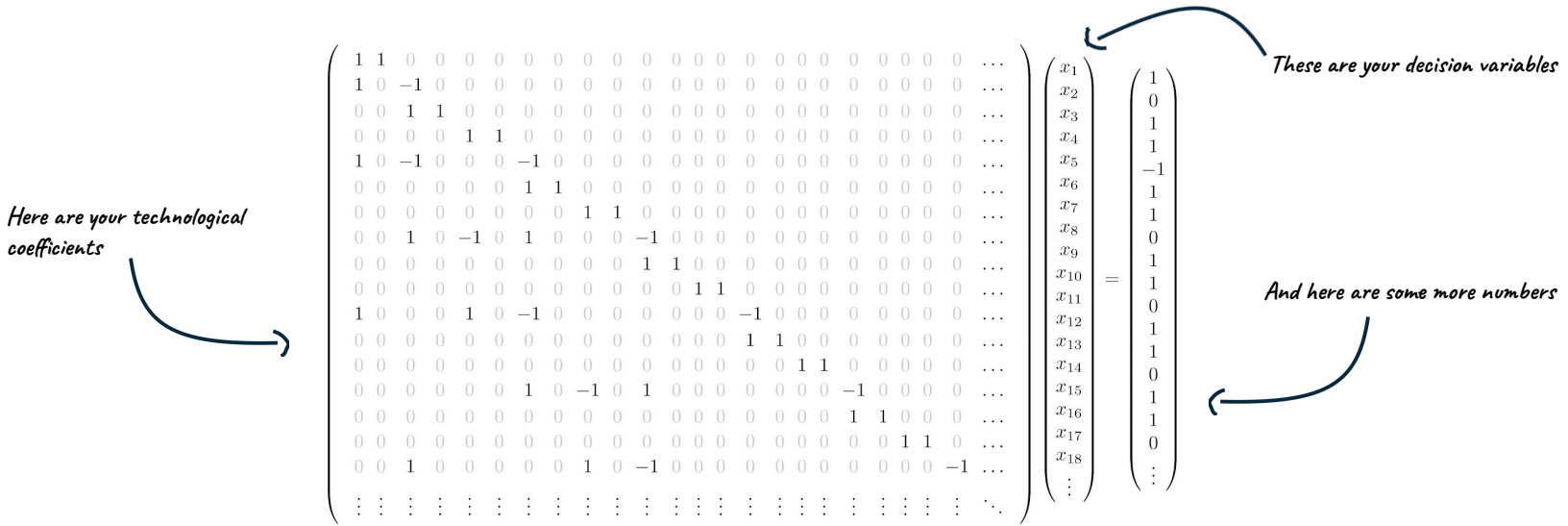
📖 Once upon a time in the 1940s

We were really good at linear algebra. So we used matrices as the underlying vehicle for optimization solvers. And many of our problems looked like this:



😬 This was a lot of work for the modeler

Modelers became logic tricksters. How do I use a matrix row to add precedence or capacity constraints to a TSP? Or represent either/or logic?



🤔 As time passed, we gained some perspective

What is optimization?
Optimization is the search for a minimum or maximum solution to some feasible set according to a given objective function

What does optimization do?
Optimization helps us make good decisions when many choices are available

What are learning now?
The initial structure of optimization is sort of a historical accident, and optimization has less to do with math as we've traditionally been taught.

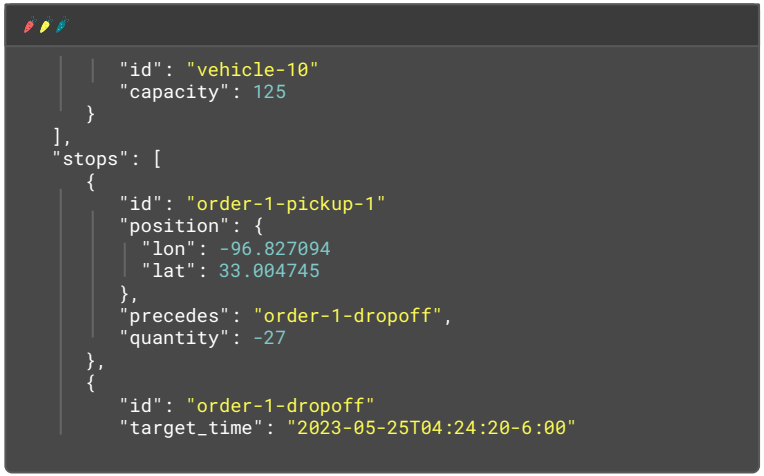
💡 Optimization is a series of strategies for evaluating and making decisions

Pulling from John Hooker's framework, nearly all successful approaches to solving large or difficult problems are based on three elements.

1. **Search:** Enumerate possible solutions.
2. **Inference:** Learn as we explore.
3. **Relaxation:** Simplify hard problems.

🧐 In the now times for decision and OR ops

Our data looks more like this:



```
      "id": "vehicle-10"
      "capacity": 125
    }
  ],
  "stops": [
    {
      "id": "order-1-pickup-1"
      "position": {
        "lon": -96.827094
        "lat": 33.004745
      },
      "precedes": "order-1-dropoff",
      "quantity": -27
    },
    {
      "id": "order-1-dropoff"
      "target_time": "2023-05-25T04:24:20-6:00"
```

But our process looks like this:

- Translate business rules to linear inequality systems
- Hand off to a solver
- ✌️🙏
- Translate solutions back to business rules

So if we think about optimization less as math and more as strategy…

…can we build models in a way that's more natural to the problem we're trying to solve?

Let’s talk decision diagrams

☝ First, some acknowledgements

We are applying this work to problems in industry. A lot of amazing folks at Carnegie Mellon and other places figured out how to use DDs to solve optimization problems, including...

- David Bergman
- Andre A. Cire
- Willem-Jan van Hoeve
- John Hooker
- And others!

🤷 What is a decision diagram?

A decision diagram (DD) is a layered, directed, multigraph that represents a set of choices. The layers correspond to the order in which we make decisions.

An “exact DD” has a path from the root node to the terminal node for every solution. The arcs costs of a path sum up to its objective value.

This turns minimization (maximization) problems into shortest (longest) path problems.

🌏 Where do DDs live in the world of optimization?

MIP

Mixed integer programming

Strong optimality reasoning

"This is the best solution!"

DDs

Decision diagrams

Good at finding feasible solutions, can prove optimality

"This is a good, timely solution!"

CP

Constraint programming

Strong feasibility reasoning

"Here are solutions!"

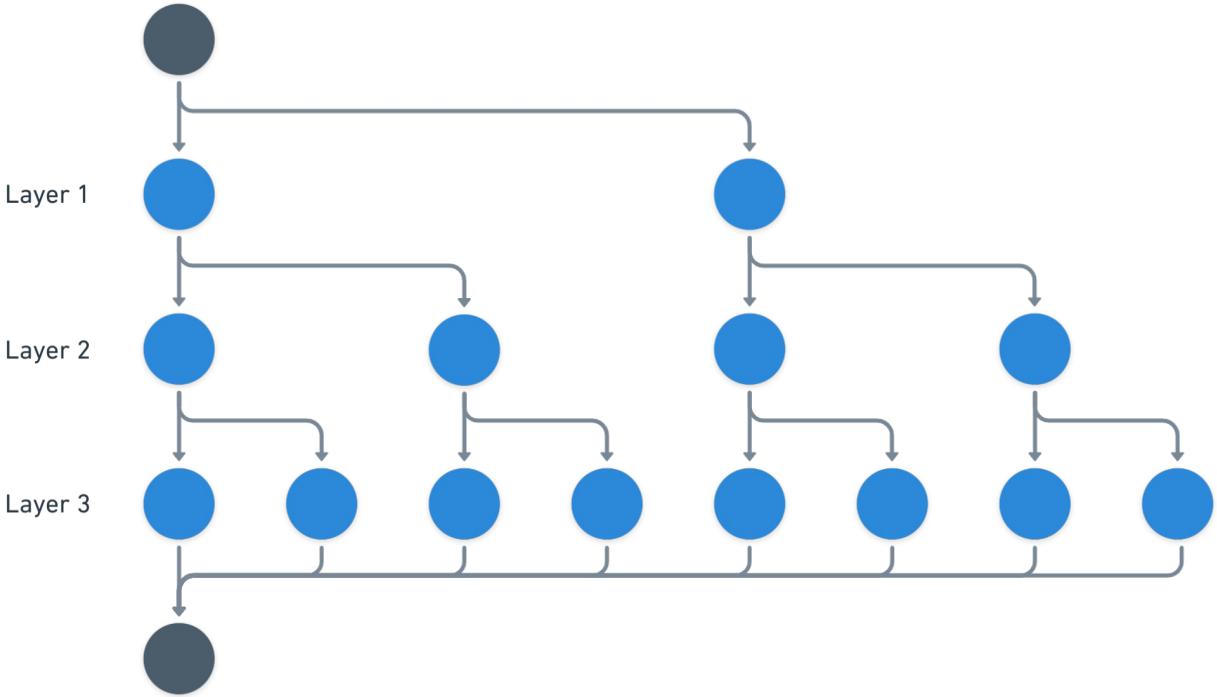
Somewhere in the middle

👀 What does a decision diagram (DD) look like?

SEARCH

Exact diagram

Show all possible states — and watch the tree get big, fast

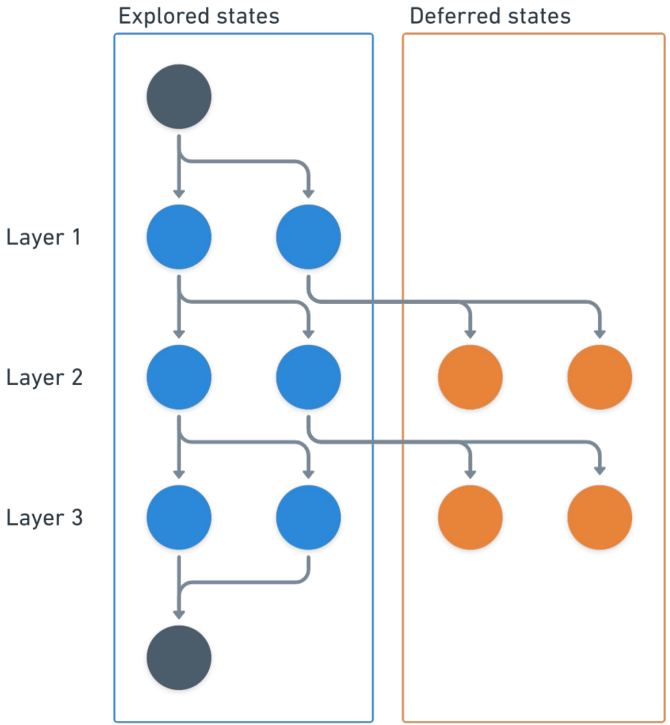


👀 What does a decision diagram (DD) look like?

SEARCH

Restricted diagram

Selectively explore some states now and some states later

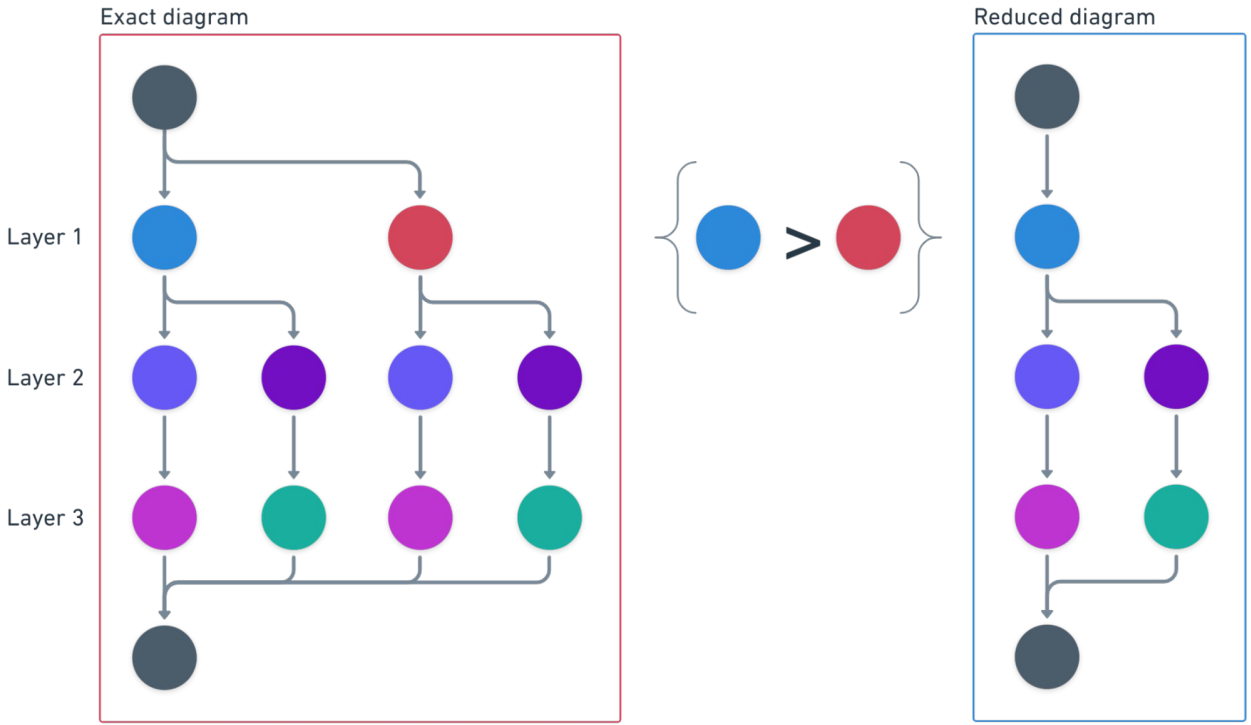


👀 What does a decision diagram (DD) look like?

INFERENCE

Reduced diagram

Learn as we explore to avoid non-productive branches of the search tree.

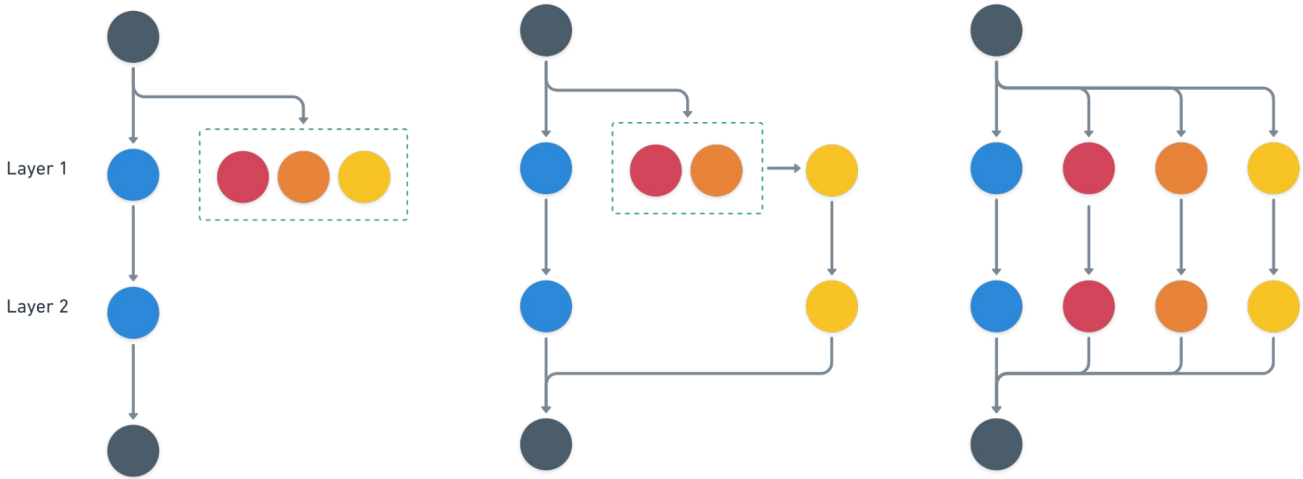


👀 What does a decision diagram (DD) look like?

RELAXATION

Relaxed diagram

Simplify hard problems by grouping states together to reduce complexity.



📖 What do we use in our DD solver?

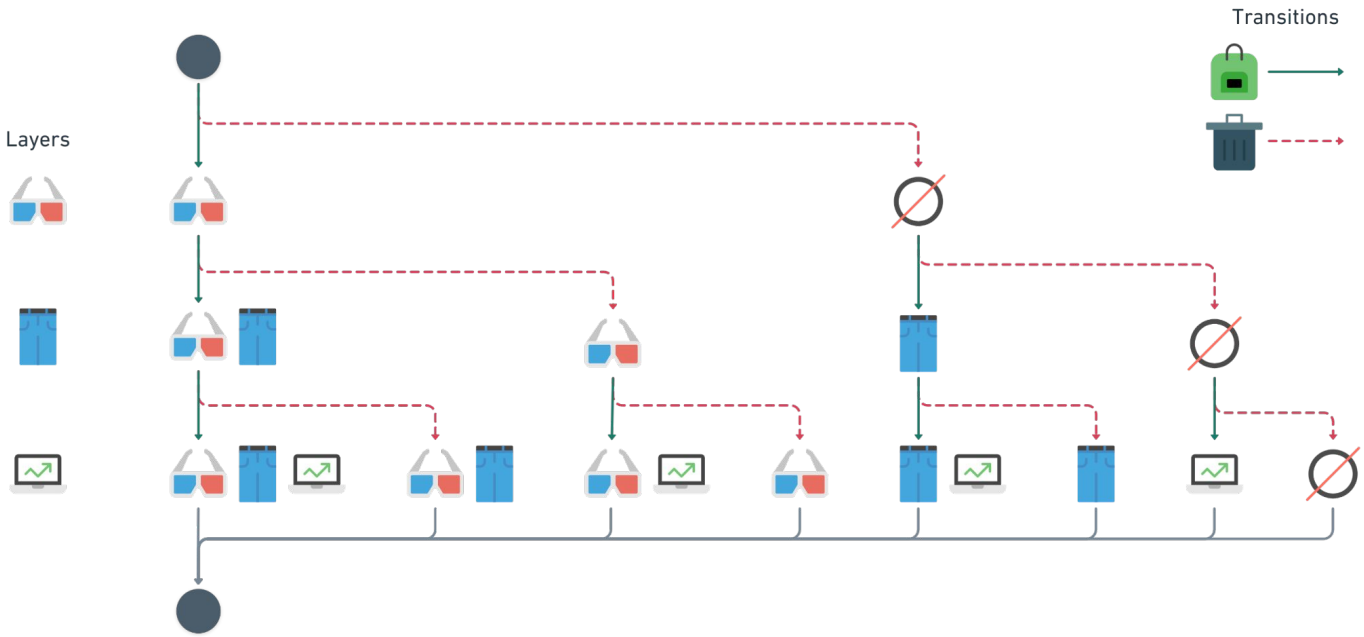
- We don’t use relaxation diagrams or merge operators. We use expanders.
- We hybridize DDs with ALNS in our routing models.
- We use reducers as our primary inference technique.

Decision diagrams in action

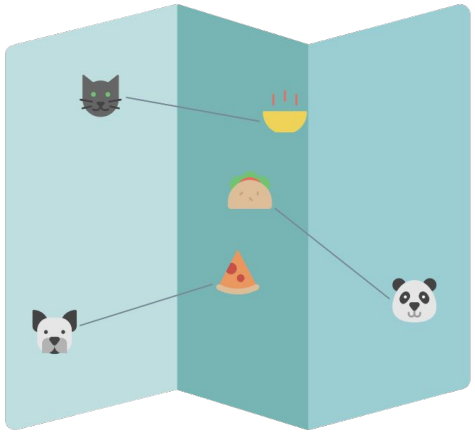
🧚 Hey! Listen! Here's what you're about to see

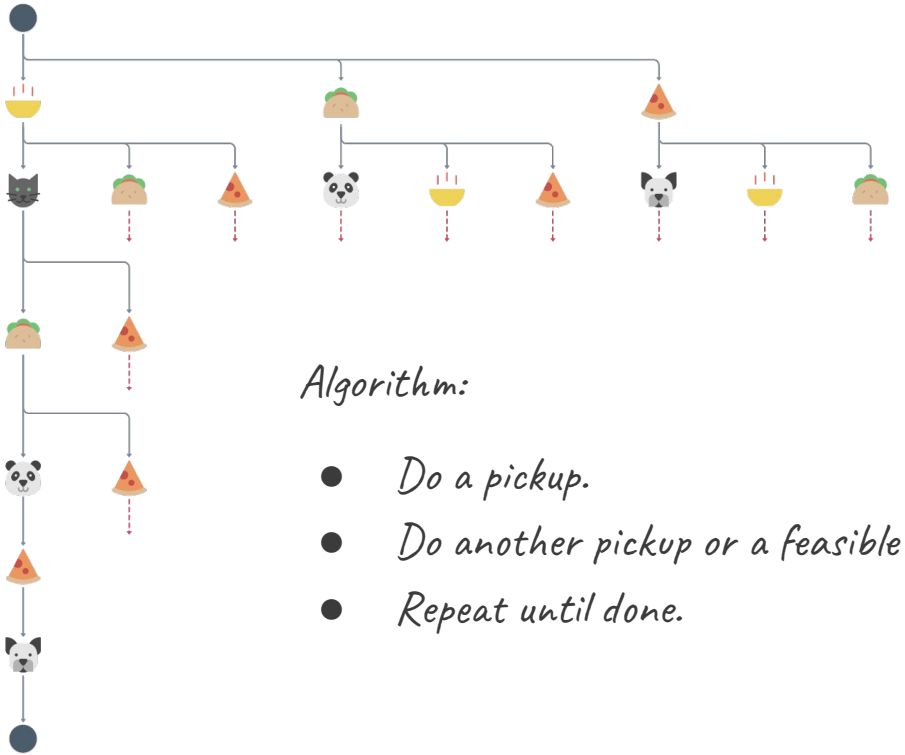
- 2 different models (knapsack and CPDTSP)
- Solve the "naive" formulation (that's the restricted diagram)
- Add bounds (through relaxation)
- Add a reducer

0-1 knapsack model



🚗 Pickup and delivery TSP





Algorithm:

- *Do a pickup.*
- *Do another pickup or a feasible delivery.*
- *Repeat until done.*

👓 Did it work? Let's look at some results

0-1 knapsack model

- Search, just search — Fully solve **dozens** of items in milliseconds!
- Search + relaxation — Fully solve **hundreds** of items in milliseconds!
- Search + relaxation + inference — Fully solve **thousands** of items in milliseconds!

Capacitated pickup & delivery TSP

- Search, just search — Optimize **6 pairs** in milliseconds!
- Search + relaxation — Optimize **8 pairs** in milliseconds!
- Search + relaxation + inference — Optimize **10 pairs** in milliseconds!

🌮 Takeaways from this talk

- Decision diagrams get us a step closer to more natural modeling for optimization problems
- They balance lots of characteristics of other solving paradigms and technologies
- We have found that they are a practical solution for real-time optimization problems

A bit of time for Q&A

Next moves

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Thank you!